

## ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E / B. Tech (Full Time) END SEMESTER EXAMINATIONS – APRIL / MAY 2024



(COMMON TO ALL)  
Semester-I  
MA7151 & MATHEMATICS-I

(Regulation 2015)

Time: 3 Hours

Answer ALL Questions

Max. Marks 100

**PART- A (10 x 2 = 20 Marks)**

Q.No	Questions	Marks
1.	Prove that $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{x}\right) = 0$ squeeze theorem	2
2.	Use intermediate value theorem to show that there is a root of the given equation in the specified interval $f(x) = e^x - 3 + 2x$ in $(0,1)$	2
3.	If $x=r\cos\theta$ and $y=r\sin\theta$ then Find the the $\frac{\partial(x,y)}{\partial(r,\theta)}$	2
4.	State Euler's theorem for homogeneous function.	2
5.	Evaluate $\int_{-1}^1 \frac{dx}{x^2}$ . If the integral does not exist justify your answer?	2
6.	What do mean by improper integral Explain	2
7.	Evaluate $\int_0^1 \int_0^2 xy \, dx dy$	2
8.	Evaluate $\int_0^\pi \int_0^{\sin\theta} r \, dr d\theta$	2
9.	Check whether the solution of the differential equation $(D^2 + 4)y = 0$ are linearly independent?	2
10.	Solve $(D^3 - 1)y = 0$	2

**PART- B (5 x 16 = 80 Marks)**

(Q. No 11 is Compulsory)

Q.No	Questions	Marks
11.	(i) Solve $(D^2 + a^2)y = \tan ax$ by method of variation of Parameters (ii) Solve: $x^2 \left(\frac{d^2y}{dx^2}\right) + 4x \left(\frac{dy}{dx}\right) + 2y = x^2 + \frac{1}{x^2}$	16

12.	<p>a)(i) Discuss the continuity at <math>x=1</math> and <math>x=3</math></p> <p>where <math>f(x) = \begin{cases} x+1, &amp; \text{if } x \leq 1 \\ \frac{1}{x}, &amp; \text{if } 1 &lt; x &lt; 3 \\ \sqrt{x-3}, &amp; \text{if } x \geq 3 \end{cases}</math></p> <p>a)(ii) Determine whether <math>f'(0)</math> exists.</p> <p><math>f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), &amp; x \neq 0 \\ 0, &amp; x = 0 \end{cases}</math></p>	16
OR		
	<p>b)(i) Find the points on the curve <math>y = x^4 - 6x^2 + 4</math> where the line is horizontal</p> <p>(ii) Discuss the curve <math>y = x^4 - 4x^3</math> with respect to concavity, points of inflection and local maxima and local minima</p>	16
13.	<p>a)(i) Find the maxima and minima of the function <math>f(x, y) = x^3 + y^3 - 12x - 3y + 20</math></p> <p>(ii) Find the Taylor's series expansion of <math>f(x, y) = e^x \cos y</math> up to third degree.</p>	16
OR		
	<p>b)(i) The Temperature <math>T</math> at any point <math>(x, y, z)</math> in the space is <math>T(x, y, z) = 400xyz^2</math>. Find the highest temperature on the surface of the unit sphere <math>x^2 + y^2 + z^2 = 1</math>.</p> <p>b)(ii) Prove that <math>\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0</math> if <math>u = \log(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)</math></p>	16
14.	<p>a)(i) Find <math>\int_0^1 \tan^{-1}(x) dx</math></p> <p>a)(ii) Prove that <math>\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx</math></p>	16
OR		
	<p>b)(i) Evaluate <math>\int \frac{2x^2 - x + 4}{x^3 + 4x} dx</math></p> <p>b)(ii) Evaluate <math>\int \frac{x}{\sqrt{x^2 + 4}} dx</math></p>	16
15.	<p>a)(i) Change the order of integration and hence evaluate it <math>\int_0^1 \int_{x^2}^{2-x} xy dx dy</math></p> <p>a)(ii) Evaluate <math>\int_0^1 \int_0^2 \int_0^3 xy^2 z dz dy dx</math></p>	16
OR		
	<p>b)(i) Find the volume of sphere <math>x^2 + y^2 + z^2 = a^2</math> by using triple integration</p> <p>b)(ii) Find the Area between two parabolas <math>y^2 = 4ax</math> and <math>x^2 = 4ay</math></p>	16

