

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)



B.E / B. Tech (Full Time) END SEMESTER EXAMINATIONS – APRIL / MAY 2024

(COMMON TO ALL)

Semester-I

MA7151 & MATHEMATICS-I

(Regulation 2015)

Time: 3 Hours

Answer ALL Questions

Max. Marks 100

PART- A (10 x 2 = 20 Marks)

Q.No	Questions	Marks
1.	Prove that $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{x}\right) = 0$ squeeze theorem	2
2.	Use intermediate value theorem to show that there is a root of the given equation in the specified interval $f(x) = e^x - 3 + 2x$ in $(0,1)$	2
3.	If $x = r \cos \theta$ and $y = r \sin \theta$ then Find the $\frac{\partial(x,y)}{\partial(r,\theta)}$	2
4.	State Euler's theorem for homogeneous function.	2
5.	Evaluate $\int_{-1}^1 \frac{dx}{x^2}$. If the integral does not exist justify your answer?	2
6.	What do mean by improper integral Explain	2
7.	Evaluate $\int_0^1 \int_0^2 xy \, dx \, dy$	2
8.	Evaluate $\int_0^\pi \int_0^{\sin \theta} r \, dr \, d\theta$	2
9.	Check whether the solution of the differential equation $(D^2 + 4)y = 0$ are linearly independent?	2
10.	Solve $(D^3 - 1)y = 0$	2

PART- B (5 x 16 = 80 Marks)

(Q. No 11 is Compulsory)

Q.No	Questions	Marks
11.	(i) Solve $(D^2 + a^2)y = \tan ax$ by method of variation of Parameters (ii) Solve: $x^2 \left(\frac{d^2y}{dx^2}\right) + 4x \left(\frac{dy}{dx}\right) + 2y = x^2 + \frac{1}{x^2}$	16

12.	<p>a)(i) Discuss the continuity at $x=1$ and $x=3$</p> <p>where $f(x) = \begin{cases} x+1, & \text{if } x \leq 1 \\ \frac{1}{x}, & \text{if } 1 < x < 3 \\ \sqrt{x-3}, & \text{if } x \geq 3 \end{cases}$</p> <p>a)(ii) Determine whether $f'(0)$ exists.</p> $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$	16
	OR	
	<p>b)(i) Find the points on the curve $y = x^4 - 6x^2 + 4$ where the line is horizontal</p> <p>(ii) Discuss the curve $y = x^4 - 4x^3$ with respect to concavity, points of inflection and local maxima and local minima</p>	16
13.	<p>a)(i) Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 12x - 3y + 20$</p> <p>(ii) Find the Taylor's series expansion of $f(x, y) = e^x \cos y$ up to third degree.</p>	16
	OR	
	<p>b)(i) The Temperature T at any point (x, y, z) in the space is $T(x, y, z) = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.</p> <p>b)(ii) Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ if $u = \log(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)$</p>	16
14.	<p>a)(i) Find $\int_0^1 \tan^{-1}(x) dx$</p> <p>a)(ii) Prove that $\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$</p>	16
	OR	
	<p>b)(i) Evaluate $\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$</p> <p>b)(ii) Evaluate $\int \frac{x}{\sqrt{x^2 + 4}} dx$</p>	16
15.	<p>a)(i) Change the order of integration and hence evaluate it $\int_0^1 \int_{x^2}^{2-x} xy dx dy$</p> <p>a)(ii) Evaluate $\int_0^1 \int_0^2 \int_0^3 xy^2 z dz dy dx$</p>	16
	OR	
	<p>b)(i) Find the volume of sphere $x^2 + y^2 + z^2 = a^2$ by using triple integration</p> <p>b)(ii) Find the Area between two parabolas $y^2 = 4ax$ and $x^2 = 4ay$</p>	16

